IDENTIFICATION OF FLEXSPLINE PARAMETERS AFFECTING THE HARMONIC DRIVE LOST MOTION USING THE GLOBAL SENSITIVITY ANALYSIS

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Resume

The aim of this research was to find out the specific geometrical parameters of the harmonic drive with the greatest impact on its lost motion. Harmonic drive units are widely used in production automation, robotics and automated manufacturing systems. Virtual model of Harmonic Drive was examined using the finite element method. The analyses were performed with considering the real test conditions of high precision gearboxes with a lost motion value of 4 % of the nominal output torque. The analyses' results were then used to develop a unique approach to obtain a functional relationship between the input parameters defining the shape of the flexspline and the harmonic drive lost motion value. Subsequently, the global sensitivity analysis was performed to determine the degree of influence of the selected flexspline parameters on the harmonic drive kinematic accuracy.

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1 Introducion

In 1955, American scientist Clarence Walton Musser published the concept of a new transmission system called „Strain Wave Gearing“. This transmission system was based on a differential transmission with a spur gear, in which the engagement was achieved by periodic elastic deformation of one of the components [1]. Harmonic drives (HD) are distinctive particularly in their compact dimensions, transmission of high torques, small lost motion and high positional accuracy. They retain those properties, especially at steady state at constant speed. Outside the steady state, the mechanisms show a non-linear characteristics, which is manifested by a significant kinematic error in operation of these systems. The application of these transmissions depends on the various advantages of harmonic transmissions. Some applications require zero lost motion and high positional accuracy, others high torque to weight ratio [2]. Experimental measurements of Ghorbel et al. [3] allowed understanding the variation of the kinematic error, depending on the load, the angular velocity and the relative position of certain components of the mechanism. With development of the computer technology, more detailed options enabled examining the properties and dependences of the system characteristics on the mechanical properties of individual components. Further studies have investigated the stress distribution in the flexspline (FS) [4]. Other studies have dealt with the effects of the FS shape on the overall HD properties. Juan et al. [5] have dealt with the optimization of the FS shape using the LSGA program in terms of weight reduction and overall HD size. Zou et al. [6] analyzed the stress and strain of the short FS at different loads.

The harmonic drive consists of three basic components that are essential for the correct operation of the transmission. The wave generator (WG) is designed most often as a thin ball bearing pressed on an elliptical cam. The circular spline (CS) is a rigid

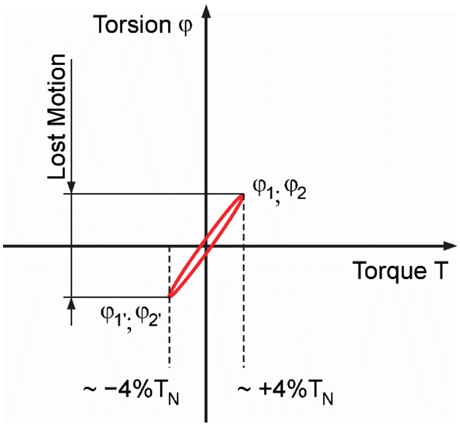


***Figure 1** Torsion characteristics of the HD by applying the ± 4% of the nominal output torque TN*

gear wheel with internal gearing. The flexspline (FS) is the most important part of the HD, which is subject to periodic elastic deformation. It has the shape of a semi-closed cylinder with external gearing at the open end. The overall properties of the HD depend mainly on properties of the WG and FS.

The aim of the presented paper was to determine the influence of selected geometrical FS parameters on the resulting kinematic accuracy, the lost motion. In the case of optimization, several design parameters of the created virtual HD model that contribute to the torsional stiffness and subsequently to the lost motion, are selected. To improve the efficiency of the optimization and design process, the process can be accelerated by eliminating the insignificant input parameters. Identification of the significant and insignificant input parameters is performed using the global sensitivity analysis (GSA), which reduces the number of input variables of the optimization problem as described by Saltelli et al. [7]. The unique method developed by Hrcek et al. [8], which identifies the key parameters of various HD components influencing its lost motion, was already well tested and described.

2 Backlash, lost motion and HD virtual model

The backlash and the lost motion terms are frequently used interchangeably in the technical literature. It is crucial to understand the distinction between the backlash and a lost motion, since the lost motion is one of the most common reasons of the positional inaccuracy in a motion system. Backlash is actually one of the components of the lost motion. It describes the state when the input torque to the mechanism does not bring any corresponding response at the output. Lost motion is a value that is determined by measuring the angular deformation in both directions with the WG fixed and CS blocked and the FS loaded with a certain torque.

Figure 1 shows the hysteresis curve obtained by the HD unit test. It records the torsional stiffness of a harmonic drive with a fixed input. Its value changes depending on the change of the transmitted load. The nonlinear torsion characteristics arises due to the thin-walled construction of the FS. It is important because of the elastic elliptical deformation occurring below the area of the outside FS gearing during the operation of the harmonic transmission system.

With increasing load, an increasing number of teeth engages due to the elastic deformation of the FS and the torsional stiffness increases. The number of engaged teeth changes while the HD gearbox is running. In addition to the torsional characteristics, related to the variable number of teeth, the characteristics in Figure 1 also depends on the elastic deformation of the FS cylindrical thin-walled part by the action of torque. The measurement of the torsional stiffness of the harmonic drive takes place with a fixed WG, using a rotation of FS in both directions with the ± 4% of the nominal output torque T_N.

Calculation of the torsional stiffness constant, which expresses the dependence of the rotation on the value of the applied torque can be expressed as

$$C = \frac{T}{\varphi}, \tag{1}$$

where:
T (Nm) - applied torque;
φ (rad) - angular displacement.

The lost motion is defined as the angular delay of the output shaft upon changing the direction of rotation of the input shaft, while in the working condition. It follows from the definition that during the measurement, the input shaft is driven by torque and the backlash is measured on the output shaft. Furthermore, the term "in the working condition" means that the output shaft is under the torque load. The analysis of harmonic transmission revealed that the lost motion is caused not only by the backlash, but by elastic deformation, as well [9].

The lost motion of the harmonic transmission is caused by backlash in the gearing between the circular spline and flexspline, as well as by the backlash in the

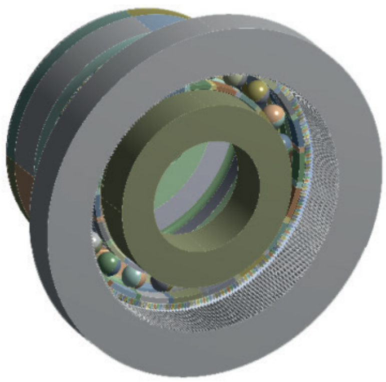


Figure 2 *The geometry of the tested HD unit*

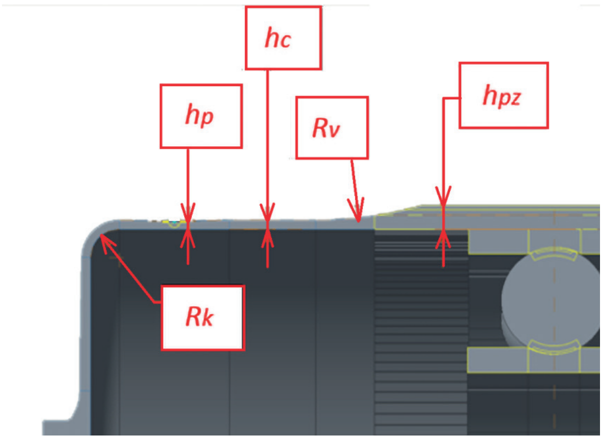


Figure 3 *Selected relevant FS dimensions affecting the HD kinematic accuracy presented on the 3D virtual mode*

Oldham coupling of the Wave generator. The source of backlash caused by elastic deformation is the flexspline under the torque load. The main causes of the backlash in the harmonic drive include [9]:

- Error in machining of the flexspline and circular spline gearing;
- Error in machining of the cam profile;
- Incorrect assembly of the three main components of the harmonic drive;
- Coupling backlash;
- Elasticity of flexspline and other parts.

The lost motion is defined with a nonlinear function that includes the backlash effect as well as the torsional characteristics of the engaging teeth with varying loading torque. This function approximates the real HD torsion characteristics and it is described by the equation [10]

$$f(\delta)=\begin{cases}\delta-b_s,\delta\geq b_s\\ \quad 0,-b_s\leq\delta\leq b_s,\\ \delta+b_s,\delta\leq b_s\end{cases} \tag{2}$$

where the backlash equals to 2 b_s.

The loss of contact between the teeth is a phenomenon that occurs especially in systems that are loaded with little or no tor que. Loss of contact and subsequent contact are a source of shocks that cause noise and strong vibrations. These in turn affect the efficiency, wear and reduce the overall life of the mechanism [11]. The kinematic accuracy is defined as the sum of the positive and negative differences between the theoretical and actual rotation output angle. The kinematic accuracy of the examined HD is less than 1 angular minute (arc min) without load and decreases to about 30 angular seconds (arc secs) with loaded gear.

In order to perform the FEM analysis an HD virtual model displayed in the Figure 2 was build following the description published by Majchrak et. al., where the finite element mesh is also described [12-14].

The resulting lost motion is influenced by many geometric parameters of the FS. The selected ones are:

- thickness under teeth $x_1 = h_{pz}$ (mm),
- thickness of cylindrical part of FS $x_2 = h_c$ (mm),
- lead radius of the gearing $x_3 = R_v$ (mm),
- end radius of cylindrical part $x_4 = R_k$ (mm),
- depth of the undercut $x_5 = h_p$ (mm).

The selected relevant FS dimensions are shown in Figure 3.

The lost motion of the HD virtual model was calculated in the ANSYS Workbench software. The material of the harmonic drive components was steel with a Young's modulus of E = 210,000 MPa and a Poisson's ratio of 0.3. Each harmonic driving component must be strained only in the elastic region; therefore, it is enough to use a linear material model where Hooke's law can be employed, as described in [8]. The nonlinear studies were performed since all the contact conditions between the HD components were determined by the Frictionless type of contact. No bonded contact type was considered.

3 Global sensitivity analysis

In order to define the most important parameters from the chosen FS dimension set, which most affect the lost motion, it is necessary to use appropriate techniques for calculating the decision-making values referring to the required variable. The global sensitivity analysis was used in this research to identify the relevant FS dimensions usable by its further optimization with accent on the HD lost motion minimization.

3.1 Functional dependence definition

To be able to express the dependence between individual changing parameters of the FS and the value of the lost motion in more detail using GSA, one first needs to know the functional dependence between the output (HD lost motion) and the input parameters (FS dimensions), $y = f(x_1, x_2, x_3, x_4, x_5)$.

To get a better picture of the dependence between the inputs parameters (iPx - selected dimensional parameters) and the output parameter (oP - lost motion) the linear regression analysis was performed. The red curve in Figure 4 interprets linear interpolation between the lost motion and the spesific dimension parameter. The black curve interprets the exponentional interpolation. The steepness of the red curve defines a sensitivity dependance between the input and the output parameters. The most popular method for solving the task is a simple linear regression. The general form is:

$$oP_{(i)} = b_0 + \sum_{j=1}^{n} b_j iP_j^{(i)}, \quad (3)$$

where the coefficients are b_0, b_j determined by the least square computation, based on the squared differences between the y-values produced by the regression model

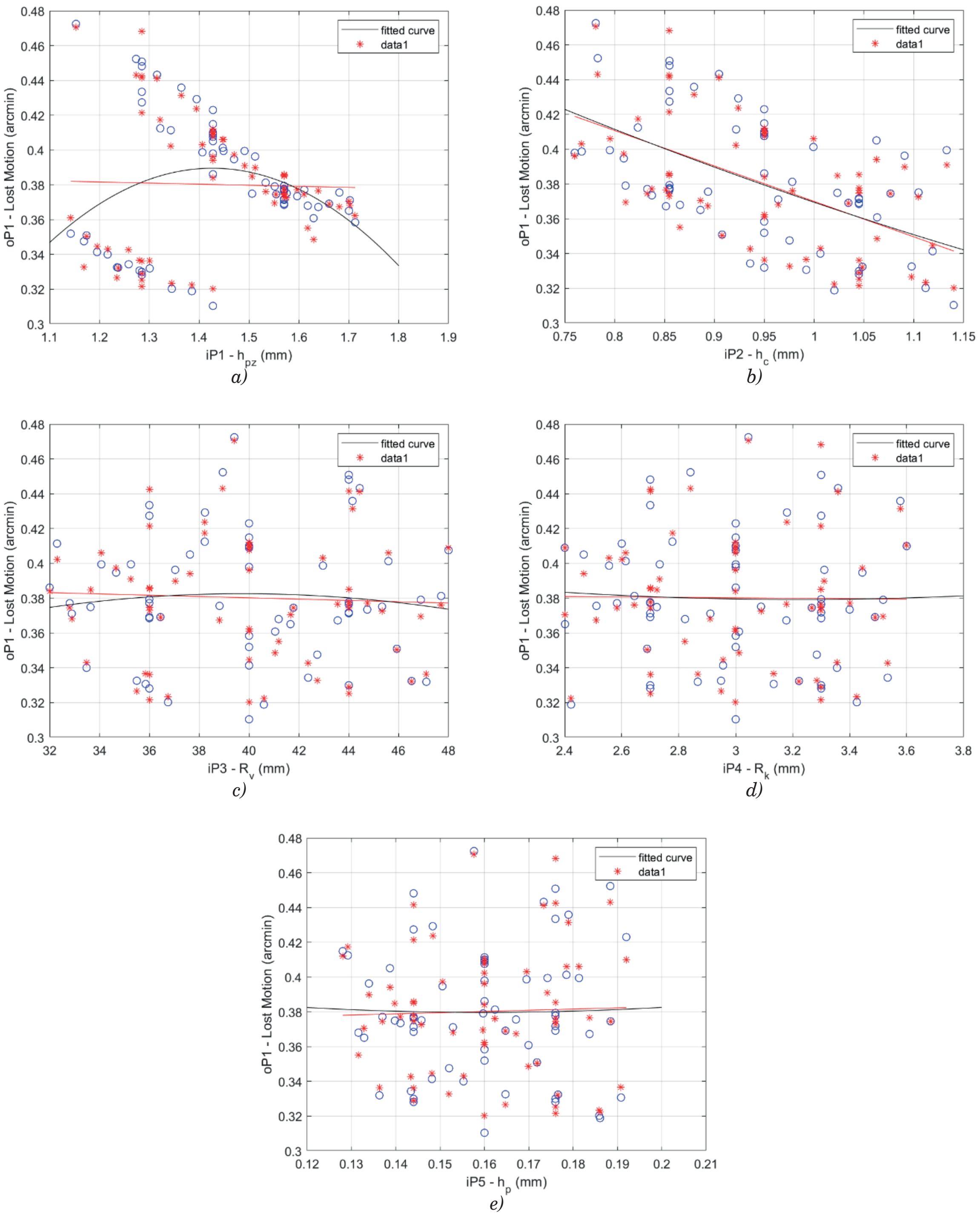
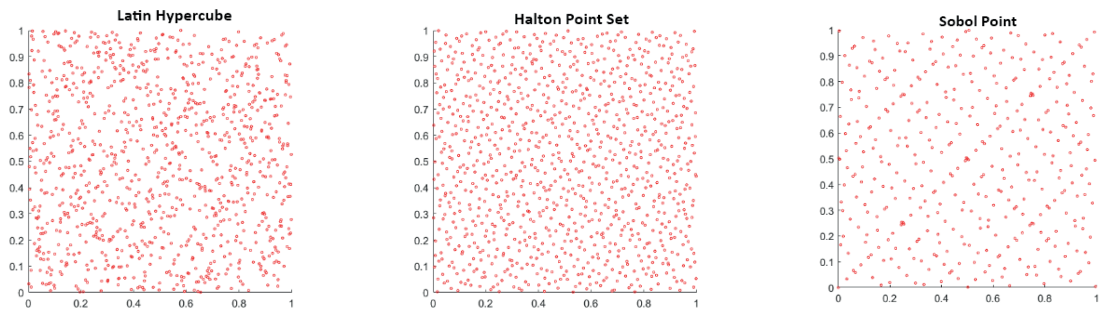


Figure 4 *Comparison of the FEM analysis lost motion results (blue circles) and the lost motion values calculated by the expressed functional dependence (red asterisks): (a) Thickness under gearing; (b) Thickness of the cylindrical part of FS; (c) Lead radius of the gearing; (d) End radius of the cylindrical part; (e) Depth of the undercut*



a) b) c)

Figure 5 *Quasi random variables scatter plots: (a) Latin Hyper Cube; (b) Halton Point Set; (c) Sobol Point*

and the actual model output obtained from the FEM analysis data. For verification, the quadratic coefficient of determination was calculated as well. The calculated value R^2 = 0.96 indicates a very good match between the data from the FEM analysis and the calculated data. The FEA results and the fitted curve function results comparison is shown in Figure 4.

3.2 Generating random numbers

After the functional dependence was obtained according to Equation (3) and proved by the quadratic coefficient of determination value, the task of a big set of the input and output variables arised in order to perform the GSA. It would be extremely time consuming to create such a big set of input and output values by the FEM, therefore the generation of the random numbers, based on former calculations, was carried out.

There are many ways to generate as random numbers as possible, but in some cases entire randomness of numbers is actually not desirable. For some types of applications, it is much more important to achieve as even random coverage of the sampled integration domain as possible, than being the numbers truly random. One of the problems with the random sequence of numbers is that the numbers can form clusters in some areas. The overall coverage of the interval may be uneven. On the other hand, the problem of a completely uniform random coverage of the interval is the possible creation of a completely uniform grid of points, which is also undesirable in some cases [15].

Quasi random sequences of numbers represent a compromise between these extremes. These sequences cannot be considered random, as they are deterministic sequences that are designed to provide as even coverage of the sampled interval as possible for a given sample and at the same time to look "sufficiently random". The individual elements of quasi random sequences deliberately avoid each other in order to avoid the clusters that we observe in random sequences [15]. Several methods for selecting the quasi random numbers are known. In this case, we selected three methods that met the requirements of this research:

- Latin Hypercube;
- Halton Point Set;
- Sobol Point.

The graphical representations of the generated random points (scatter plots) according to these methods are shown in Figure 5.

The calculated values of the lost motion for the individual input parameters from the expressed function, based on the quasi random variables scatter plot by Sobol Point, are shown in Figure 6. The cyclic procedure of their calculation started with a small number of the quasi random variables, which was gradually increased until the convergence was reached. The Sobol Point method was chosen due to the possibility of solving higher order interaction of the input parameters. In order to calculate the Sobol indices, it is necessary to include a large number of data sets in the range of 1000 - 10000 in the analysis. Due to the better comprehensibility of the graphs, only thirteen of them are displayed. The data sets are generated from the detected dependencies of individual input parameters on the output parameter from the FEM analysis.

3.3 Sobol indices calculation

To generate the quasi random data for further GSA, the Sobol Point method was used. The advantage of this method was the speed of data generation. We know several Sobol´s indices [16], but for our calculation the first-order sensitivity index and the total-order index are important. Let the generic model, described in more detail in [7], be used:

$$Y = f(X_1, \ldots, X_p). \tag{4}$$

The output Y is a scalar, while the input elements X_p,..., X_p are independent random variables with known probability distributions. Those distributions reject the system's uncertain knowledge. The primary idea behind this strategy is to break down the output variation into contributions from each input factor. Derivation of the relation for calculation is described by Homma and Saltelli [17]. Using the law of total variance, one can write

$$V(Y) = V_{Xi}(E_{X\sim i}(Y|X_i)) + E_{Xi}(V_{X\sim i}(Y|X_i)), \quad (5)$$

where V is a expected value of process variance and E is a variance of the hypothetical means. The first-order influence of X_i on Y is known as the conditional variance $V_{Xi}(E_{X\sim i}(Y|X_i))$ and the sensitivity measure

$$S_i = \frac{Vx_i(E_{X\sim i}(Y|X_i))}{V(Y)} \quad (6)$$

is known as the first-order sensitivity index of X_i on Y. S_i. S_i is a number always between 0 and 1. The total-order sensitivity index, ST_i described to account for all contributions to output variation owing to factor X_i (i.e. the first-order index plus all its interactions):

$$ST_i = \sum\nolimits_{k\neq 1} S_k, \quad (7)$$

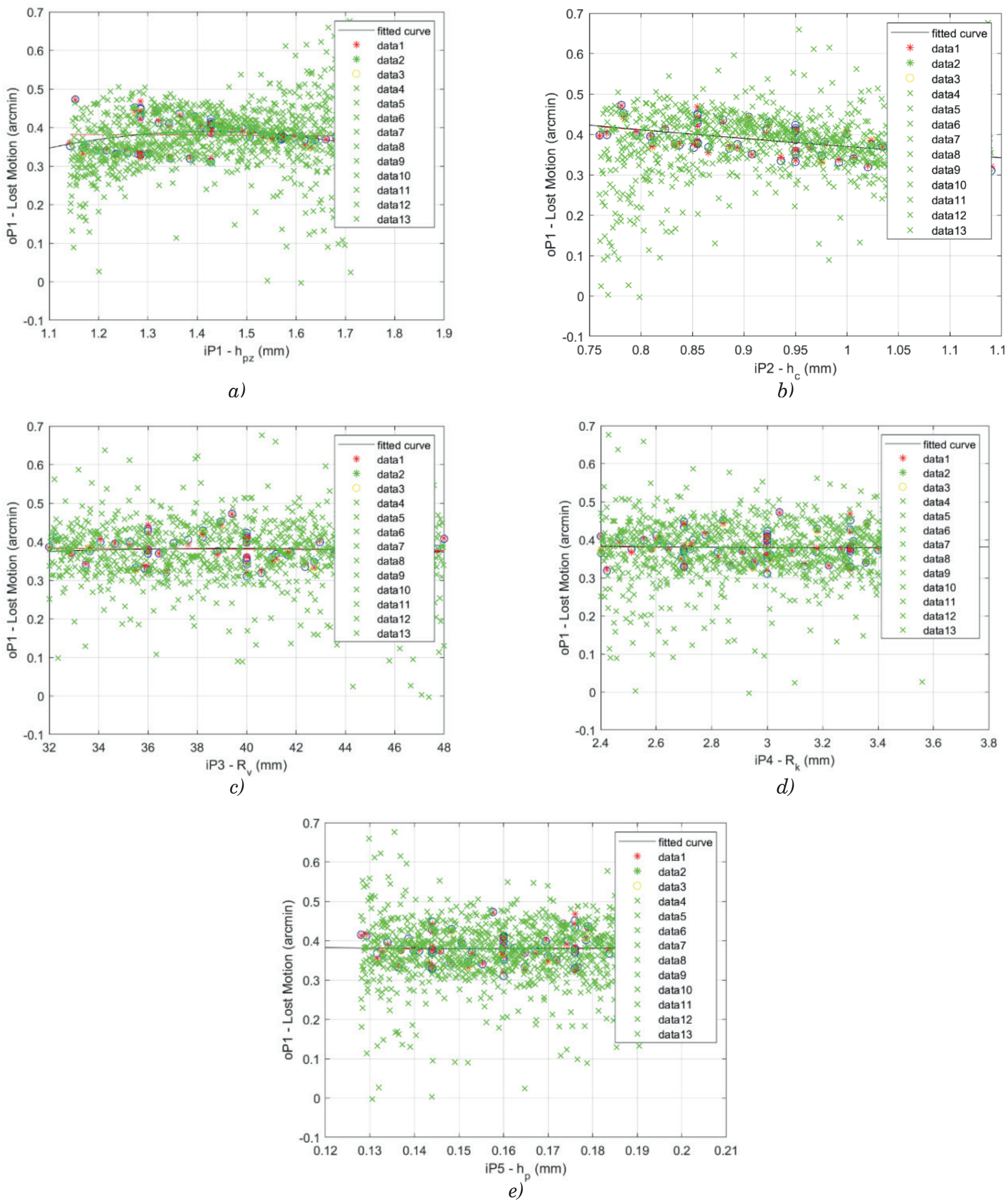


***Figure 6** The lost motion values of the fitted function calculated for the Sobol Point data set: (a) Thickness under gearing; (b) Thickness of the cylindrical part of FS; (c) Lead radius of the gearing; (d) End radius of the cylindrical part; (e) Depth of the undercut*

where $k \neq 1$ indicates all the indices associated to the factor X_i. The total-order index can be expressed as

$$ST_i = 1 - \frac{Vx \sim_i (E_{Xi}(Y|X_{\sim i}))}{V(Y)} \tag{8}$$

and using again the law of total variance and normalizing we get

$$1 = \frac{Vx \sim_i (E_{Xi}(Y|X_{\sim i}))}{V(Y)} + \frac{Ex \sim_i (V_{Xi}(Y|X_{\sim i}))}{V(Y)}. \tag{9}$$

The second term in equation (9) is the total-order index [15]

$$ST_i = \frac{Ex \sim_i (V_{Xi}(Y|X_{\sim i}))}{V(Y)}. \tag{10}$$

The graphically illustrated influence of individual input parameters with respect to the output parameter using GSA can be seen in Figure 7. The First order is represented by the red columns and the Total effect by the blue columns. The resulting values of Sobol's indices are presented in Table 1.

4 Finding and discussion

The presented research dealt with the analysis of the lost motion in a harmonic drive. depending on changing some design parameters of the harmonic drive flexspline. Based on the knowledge from previous research focused on the influence of various HD parameters on its lost motion. The flexspline was selected as the key component of the HD unit. Based on the experience gained, five key geometric parameters, defining the FS geometry with the highest impact on the lost motion, were selected. The examined gearbox was loaded with ± 4 % of the rated output torque, which simulated the real conditions of high precision harmonic drives for robotic and automated manufacturing system applications.

The calculated First-order and Total-effect Sobol's indices in Table 1 indicate that the thickness under the gearing h_{pz} and the thickness of the cylindrical part of FS h_c have the greatest affect to the lost motion. The above mentioned results are consistent with the assumptions based on previous experience with harmonic drive unit design. These identified parameters had the greatest impact on the lost motion even in prototype tests.

The results obtained using the method based on the GSA developed by Hrcek et al. and described in detail in [8] allow to identify unambiguously the relevant parameters affecting the HD kinematic accuracy not only by parameters of various HD components but by various parameters of a the flexpline, as well. The results can help by optimizing the FS and thus the whole HD unit. Based on the results of the used method based on the GSA, the entire optimization process can be accelerated by eliminating irrelevant parameters and thus reducing the number of variants needed for the time consuming finite elements analyses.

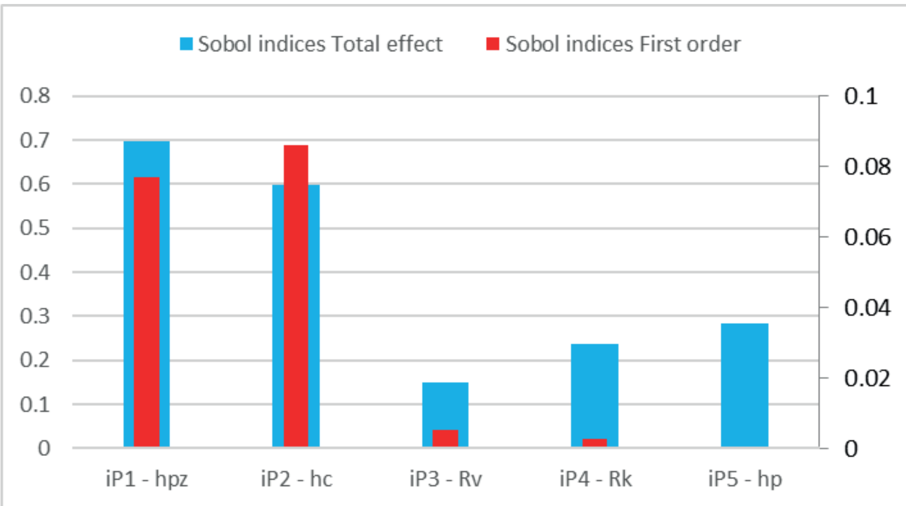


Figure 7 *Results of calculating the Sobol's first-order and total-effect indices*

Table 1 *Values of the calculated Sobol's indices*

FS dimension		iP1 (h_{pz})	iP2 (h_c)	iP3 (R_v)	iP4 (R_k)	iP5 (h_p)
Correlation coefficient	Sobol indices First order	0.0771500	0.0859400	0.0052840	0.0027370	0.0001973
	Sobol indices Total effect	0.6969000	0.5974000	0.1497000	0.2385000	0.2825000

5 Conclusion

The main contribution of the presented research is the clear identification of the input dimensional parameters with the greatest influence on the lost motion. So far, such a precisely quantified dependence has not yet been published in scientific publications. The findings can be used in the design process of harmonic gearboxes. The early stage design process can be directly focused on the specific dimensional parameters with the greatest impact on the lost motion. On the the other hand, less attention can be paid to parameters with less impact. Further research will be focused on investigating the influence of other dimensional parameters on the kinematic properties of harmonic gearboxes. Another goal will be to investigate the torsional properties of individual componets of harmonic gearboxes, as well as the transmission system as a whole unit.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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